Lecture 18 Summary

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Superconductors in a Magnetic Field - H_{c2} and Vortices
We see how superconductivity nucleates in a strong magnetic field.

0.1 Linearized GL Equation

We consider the onset of superconductivity for $T < T_c$ in a strong magnetic field as the field is reduced. We look for the onset of superconductivity under this demanding situation in which $|\psi|^2 << |\psi_\infty|^2$. In this case we can neglect the $\psi|\psi|^2$ term in the GL equation to linearize it:
$\alpha\psi + \frac{1}{2m^*}\left(\frac{\hbar}{i}\overrightarrow{\bigtriangledown} - e^*\overrightarrow{A}\right)^2\psi = 0.$
We can also write this linearized GL equation as
$\left(\frac{1}{i}\overrightarrow{\bigtriangledown} - \frac{2\pi}{\Phi_0}\overrightarrow{A}\right)^2\psi = \frac{1}{\xi_{GL}^2}\psi.$
The expression for the current density $\overrightarrow{J} = \frac{e^*}{m^*}|\psi|^2\left(\hbar\overrightarrow{\bigtriangledown}\phi - e^*\overrightarrow{A}\right)$ decouples from the linearlized GL equation if we assume that the total vector potential $\overrightarrow{A}$ has contributions from the external vector potential only. In other words the order parameter is so weak that we can neglect the screening produced by the superconductor.

The linearized GL equation is a Schrodinger equation with eigenvalue $\epsilon = -\alpha$. There will be an infinite number of such eigenvalues, which correspond to temperatures,
$\epsilon_j = -\alpha_j = \alpha'(1 - \frac{T_j}{T_c}) = \alpha'(1 - t_j)$. The smallest eigenvalue will correspond to the largest temperature for a solution, $t_j = 1 - \frac{\epsilon_j}{\alpha'}$. This will result in the prediction of the superconductor/normal phase boundary.

0.2 Calculation of H_{c2} in a Bulk Superconductor

Consider an infinite superconductor in a strong external magnetic field $\overrightarrow{H} = H\widehat{z}$. This can be represented with the vector potential $\overrightarrow{A} = \mu_0 Hx\widehat{y}$. Put this choice of $\overrightarrow{A}$ into the linearized GL equation, divide through by ψ_∞ to form $f = \psi/\psi_\infty$,

square the operator and ignore any variation of the order parameter in the direction of the magnetic field (z) to obtain,
$-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)f+\left(\frac{2\pi}{\Phi_0}\mu_0 H\right)^2 x^2 f+\frac{i4\pi}{\Phi_0}\mu_0 Hx\frac{\partial f}{\partial y}=\frac{f}{\xi_{GL}^2}$.
Taking $H=0$ yields the simple solution $f(x,y)=e^{ik_x x}e^{ik_y y}$ which has uniform magnitude. This suggests the following ansatz for the full equation:
$f(x,y)=g(x)e^{iky}$, where k will be determined later.

Substituting this yields the following second order differential equation for $g(x)$,
$\frac{-\hbar^2}{2m^*}g''+\frac{1}{2}k_s(x-x_0)^2 g=\frac{\hbar^2}{2m^*}\frac{g}{\xi_{GL}^2}=\epsilon g=-\alpha g$,
where the "spring constant" is $k_s\equiv\frac{(e^*\mu_0 H)^2}{m^*}$, and $x_0\equiv\frac{\Phi_0 k}{2\pi\mu_0 H}$.
This is the Schrodinger equation for a one-dimensional harmonic oscillator centered on $x=x_0$ with eigenvalues $\epsilon_n=(n+\frac{1}{2})\hbar\omega$ with $\omega=\sqrt{\frac{k_s}{m*}}$. The lowest eigenvalue ($n=0$) corresponds to the highest t and represents the phase boundary, $\frac{\hbar^2}{2m^*}\frac{1}{\xi_{GL}^2}=(n+\frac{1}{2})\hbar\frac{e^*\mu_0 H}{m^*}$.
Solving for the magnetic field at the phase boundary (for a given $T<T_c$) yields, $H_{c2}(T)=\frac{\Phi_0}{2\pi\mu_0}\frac{1}{\xi_{GL}^2}$.

$H_{c2}(T)$ represents the highest field at which superconductivity can nucleate for a given $T<T_c$. Note that as $T\to T_c$ ξ_{GL} diverges and H_{c2} goes to zero. From the temperature dependence of ξ_{GL} we see that near T_c one has $H_{c2}(T)\sim 1-t$. Thus from the linear slope of $H_{c2}(T)$ at T_c one can deduce the extrapolated zero-temperature GL coherence length.

One can re-write $H_{c2}(T)$ in various forms using other GL quantities as follows:

$H_{c2}=\frac{4\pi\mu_0}{\Phi_0}\lambda_{eff}^2 H_c^2$,

$H_{c2}=\sqrt{2}\frac{\lambda_{eff}}{\xi_{GL}}H_c=\sqrt{2}\kappa H_c$.
This last expression shows that $H_{c2}>H_c$ for type-II superconductors ($\kappa>1/\sqrt{2}$), and $H_{c2}<H_c$ for type-I superconductors ($\kappa<1/\sqrt{2}$). As an example, consider the cuprate superconductor YBCO which has $\mu_0 H_c\sim 1T$ and $\kappa\sim 60$. It's upper critical field $\mu_0 H_{c2}\sim 85T$.

Imagine a superconductor in a large magnetic field at $T<T_c$ such that it is a normal conductor. The magnetic field is now reduced and we examine the evolution of the GL order parameter. For a type-II superconductor the GL order parameter is zero until we reach $H=H_{c2}$ and then it increases continuously as the field is reduced (characteristic of a second order phase transition). This process is reversible and non-hysteretic. Nothing special happens as H is reduced through H_c.
For a type-I superconductor one goes through $H=H_c$ and the order parameter

remains zero. Only when the field is reduced to the lower value of $H = H_{c2}$ does superconductivity nucleate and the order parameter suddenly jumps up to its equilibrium value at that temperature and field. This super-cooling process means the superconductor is out of equilibrium. If the field is now increased, the order parameter remains large until the thermodynamic critical field H_c is reached, at which point the order parameter is discontinuously reduced to zero in a first-order transition. The entire cycle is an open hysterisis loop, characteristic of first-order phase transitions.